Orszag Tang vortex - Kinetic study of a turbulent plasma

T. N. Parashar, S. Servidio, M. A. Shay, W. H. Matthaeus and P. A. Cassak[1]

Department of Physics & Astronomy, 217 Sharp Lab, University of Delaware, Newark, Delaware 19716, USA

Abstract. Kinetic evolution of the Orszag-Tang vortex is studied using collisionless hybrid simulations based on particle in cell ions and fluid electrons. In magnetohydrodynamics (MHD) this configuration leads rapidly to broadband turbulence. An earlier study [1] estimated the dissipation in the system. A comparison of MHD & hybrid simulations showed similar behavior at large scales but substantial differences at small scales. The hybrid magnetic energy spectrum shows a break at the scale where Hall term in the Ohm's law becomes important. The protons heat perpendicularly and most of the energy is dissipated through magnetic interactions. Here, the space time structure of the system is studied using frequency-wavenumber (k-ω) decomposition. No clear resonances appear, ruling out the cyclotron resonances as a likely candidate for the perpendicular heating. The only distinguishable wave modes present, which constitute a small percentage of total energy, are magnetosonic modes.

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INTRODUCTION

Kinetic dissipation in the corona and solar wind is central to understanding of the sun and the heliosphere. Coronal heating, solar wind heating, and many other astrophysical problems require an understanding of the mechanisms that convert energy from fluid motions and magnetic fields into thermal energy. In relation to the solar wind, the open question is the heating of interplanetary plasma. The temperature of the solar wind is greater than that expected by adiabatic expansion [2]. There is also preferential heating in the direction perpendicular to the mean magnetic field (e.g. [3] and references therein).

One of the mechanisms generally used to explain anisotropic heating of protons is cyclotron damping of high frequency waves [4, 5]. One way to propagate energy to such wavenumbers has been studied in the weak turbulence limit [6, 7]. Three wave interactions between fast and Alfvén waves transfer energy to very small scales and cyclotron resonances damp this energy away. Other mechanisms involve coupling of electron phase-space holes to protons [8, 9] and heating of protons by the nonuniform electric fields in the current sheets and reconnection sites [10]. MHD simulations use some kind of viscosity and/or resistivity to dissipate energy at the smallest scales. The solar wind is almost completely collisionless [3] rendering the underlying local approximations (e.g. [11]) for viscosity and resistivity questionable. Some attempts have been made to include kinetic dissipation in the form of hyperviscosity, hyperresistivity, Hall effect, finite Larmor radius effects etc. (e.g. [12] and the references therein). Self consistent kinetic studies are necessary to understand the underlying physical processes contributing to these effects.

1D PIC simulations [13, 14], PIC simulations of sheet pinch [15], PIC simulations of Whistler turbulence [16] and gyrokinetic simulations [17] have been used to look at the issue of dissipation. Full particle simulations being computationally very expensive, we resort to hybrid simulations which are not as computationally expensive. Hybrid simulations give us the freedom of running a reasonably big system but still keeping all the kinetic effects at the proton inertial length scales.

In this paper we present results of hybrid simulations of Orszag-Tang vortex (OTV) [18]. This initial condition rapidly evolves into fully developed highly nonlinear turbulence and can be used as a test bed to approach the problem of dissipation in collisionless plasmas. The OTV was set up and studied in a 2.5 hybrid code. This system shows preferential perpendicular heating of ions and has zero effective viscosity [1]. Some studies have suggested preferential proton heating can be due to cyclotron resonances [4, 5], kinetic Alfvén waves (KAWs) [17], or other strong wave particle interactions. The simulations of this study however do not have $k_{||}$ to the mean magnetic field and hence cannot have cyclotron resonances. Furthermore the $k-\omega$ diagrams of energy do not show any significant wave-like activity. The probability density functions (pdfs) develop tails departing from Gaussianity which are a sign of strong coherent structures [19, 20]. The time evolution of this system creates intense current sheets and reconnection sites which are good contenders for being the intermittent structures characterizing turbulence and also for heating the ions

[1] Present address: Department of Physics, Hodges Hall, Box 6315, West Virginia University, Morgantown, WV 26506

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perpendicularly (e.g. [10]).

SIMULATION DETAILS & RESULTS

The problem of kinetic dissipation requires a lot of different physical length scales to be treated, all the way from MHD range to smaller than ion kinetic scales. This is very expensive to do in full particle simulations. Hybrid simulations, even though much more computationally expensive than MHD, are capable of running a big enough system to include part of the inertial range and have all the kinetic physics at the ion length scales.

The Orszag-Tang vortex [18] is given by

$$\mathbf{B} = -\sin y\,\hat{\mathbf{x}} + \sin 2x\,\hat{\mathbf{y}} + B_g\,\hat{\mathbf{z}} \tag{1}$$

$$\mathbf{v} = -\sin y\,\hat{\mathbf{x}} + \sin x\,\hat{\mathbf{y}} \tag{2}$$

It is an initial condition that rapidly evolves into strong turbulence. It is extensively used to check the stability of numerical schemes. It is a first step towards understanding the physics of collisionless turbulent plasmas.

A parallel version of the hybrid code described in [21], was used in 2.5D for the simulations. Protons are treated as particles and electrons as fluid. This code has been extensively used to simulate magnetic reconnection. The code advances the following equations:

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \tag{3}$$

$$\frac{d\mathbf{v}_i}{dt} = \frac{1}{\varepsilon_H}\left(\mathbf{E}' + \mathbf{v}_i \times \mathbf{B}\right) \tag{4}$$

$$\frac{\partial \mathbf{B}'}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \varepsilon_H \nabla \times \left(\frac{\mathbf{J}}{n} \times \mathbf{B}'\right) \tag{5}$$

$$\mathbf{B}' = \left(1 - \frac{m_e}{m_i}\varepsilon_H^2 \nabla^2\right)\mathbf{B} \tag{6}$$

$$\mathbf{E}' = \mathbf{B} \times \left(\mathbf{v} - \varepsilon_H \frac{\mathbf{J}}{n}\right) \tag{7}$$

where $\mathbf{J} = \nabla \times \mathbf{B}$ is the current density, $\varepsilon_H \equiv c/(L_0\omega_{pi})$ is the normalized proton inertial length, m_e and m_i are the electron and proton masses, $\mathbf{x}_i$ and $\mathbf{v}_i$ are the positions and velocities of the individual protons, and $\mathbf{v}$ is the proton bulk flow speed. Length is normalized to L_0, velocity to $V_0 = B_0/(4\pi m n_0)^{1/2}$, time to $t_0 = L_0/V_0$, and temperature to $B_0^2/(4\pi n_0)$. The average density is n_0, and B_0 is the root mean square in-plane magnetic field. The magnetic field $\mathbf{B}$ is determined from $\mathbf{B}'$ using the multigrid method. The fields are extrapolated to the particle positions using a first-order weighting scheme, which is essentially linear interpolation[22]. This allows the smooth variation due to particle motion of the fields felt by the particle. A similar first-order weighting scheme is used to determine the fluid moments at the grid locations from the particles. We assume quasi-neutrality and has periodic boundary conditions and zero electron temperature.

The simulation box is size $2\pi \times 2\pi$ with 512×512 grid points. About 105 million protons were loaded with a Maxwellian distribution with temperature $T_i = 8.0$ initially. Other parameters were $\varepsilon_H = 2\pi/25.6$, $m_e = 0.04m_i$, $B_g = 5.0$ to reduce the compressibility. The plasma β is ~ 0.62. No artificial dissipation was included other than the grid scale dissipation. Initial perturbations in density n_0 which enforce $\partial(\nabla\dot{\mathbf{v}})/\partial t = 0$ at $t = 0$ were added to enhance the incompressibility induced by the guide field. This exact system was also run using an MHD code for comparisons with hybrid code.

Some results are discussed in [1]. A comparison of energies and enstrophies from hybrid and MHD simulations shows that the kinetic length scales involved in the hybrid case are smaller than the MHD case. A good fraction of the free energy is converted into thermal energy with preferential heating in the perpendicular direction. During the self similar turbulent regime, most of the heating occurs through magnetic field. A comparison of the dissipation with the MHD dissipation terms also emphasizes this point, as we see zero effective viscosity and a non zero effective resistivity.

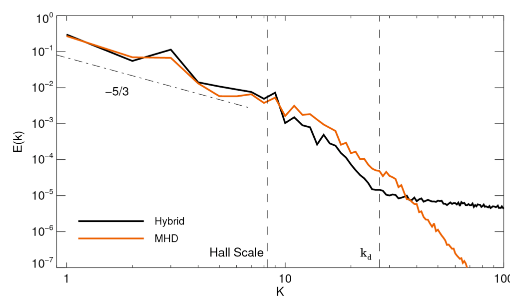

FIGURE 1. The magnetic energy spectra follow each other very closely in the MHD range but at the Hall scale, the hybrid spectrum drops below the fluid spectrum [1]. The Hall scale and MHD Kolmogorov dissipation scale (k_d) are shown for reference. The Hall scale, where dispersive kinetic Alfvén waves arise, occurs when $kd_i c_s/c_m \geq 1$, where c_s is the sound speed and c_m is the magnetosonic speed [23].

The magnetic energy spectra (Fig. 1) of hybrid and MHD simulations follow very closely in the MHD range. At the length scale at which Hall term becomes important, the hybrid spectrum drops more indicating stronger dissipation mechanism at those scales.

This paper discusses the spectra of the OTV in hybrid code in detail. Many mechanisms proposed earlier (e.g. [4, 5, 17]) involve wave particle interactions like cyclotron resonances or kinetic Alfvén waves (KAWs). The classical cyclotron resonance condition requires a $k_{||}$ to the mean magnetic field for the resonance to occur. Our system does not have a $k_{||}$ and hence there should not be any cyclotron resonances. However there could be a kind

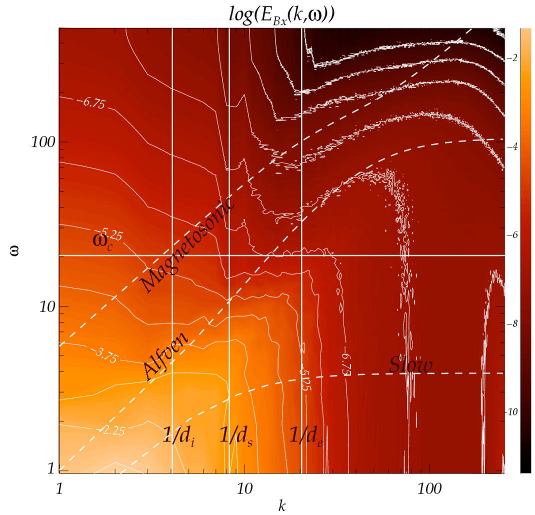

FIGURE 2. Shaded contours of energy in B_x plotted in the $k-\omega$ space on logarithmic scales. Contour lines show levels of energy. Dispersion relations are over-plotted (see [23]), calculated using a two fluid dispersion relation, using the k parallel to the in-plane magnetic field and $\perp$ to B_g. Important length and time scales are marked. The spectrum shows no significant power in wave modes.

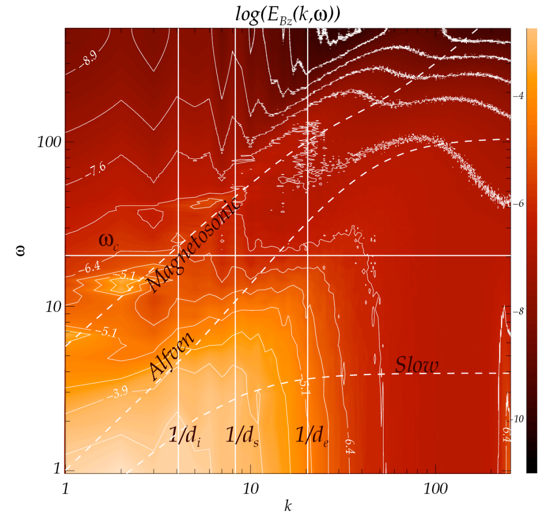

FIGURE 3. Shaded contours of energy in B_z in $k-\omega$ space. See Fig. 2 for details. This spectrum also does not show significant power in wave modes. There is slight hint of magnetosonic activity but the power in that mode is about two orders of magnitude smaller than the maximum power available.

of resonance relative to the "local mean field" when the finite amplitude fluctuations are taken into account.

To further support this argument, we look at the energy in the $k-\omega$ space. Figure 2 shows the $k-\omega$ spectrum of energy in B_x. It shows no signs of any wave like activity. The spectrum is completely featureless. The $k-\omega$ spectrum of B_z (fig. 3) has slight hint of magnetosonic activity but the energy in that mode is about two orders of magnitude smaller than the available energy in the system. There is no sign of cyclotron wave activity. The Eulerian spectrum (fig. 4) has a slope of $-5/3$ which is often discussed in hydrodynamics as a consequence of random sweeping in strong turbulence (e.g. [24, 25]. This result being valid in plasma turbulence is not necessarily straightforward. It also clearly emphasizes the small amount of power in the magnetosonic mode and the absence of cyclotron resonance.

The $k-\omega$ spectra show no obvious signatures of significant wave activity in this system. This study suggests that it is not necessary to have strong discrete frequency resonant wave particle interactions to dissipate energy in turbulent plasmas. However some form of nonlinear or non-resonant wave particle interactions are implied.

The absence of significant wave modes in our system calls for another explanation of heating. One possibility is heating caused by energization of particles as they sample current sheets and reconnection sites [10]. It would require frequent intermittent current sheets and reconnection sites for the cumulative heating to be significant.

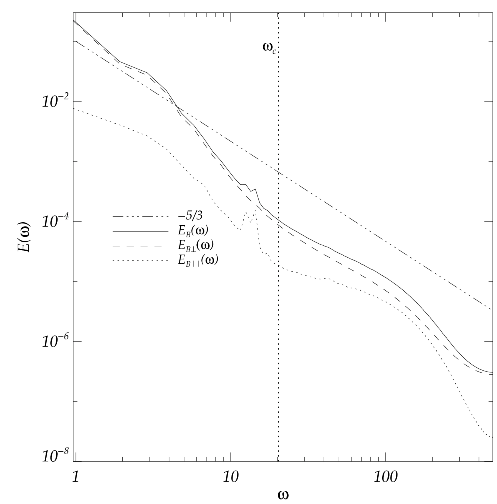

FIGURE 4. The Eulerian frequency spectrum emphasizes the point again that the only recognizable wave mode is the magnetosonic mode. Also it clearly shows that there is no power at the cyclotron frequency to drive the perpendicular heating discussed in [1]. Note the $-5/3$ slope of the spectrum too which is a property of the hydrodynamic spectrum.

The normalized PDFs of velocity and magnetic fields (fig. 5) coincide with the unit Gaussian very closely, signaling the statistically homogeneous turbulent behavior at large scales (e.g. [26]). The PDFs of velocity field vector increments as well as magnetic field vector increments, (e.g. $|\Delta\mathbf{B}| = |\mathbf{B}(s+\Delta s) - \mathbf{B}(s)|$), at points separated by Δs show departures from Gaussianity for small Δs. This is a clear sign of intermittent structures in the system [19, 20]. These intermittent structures in our system happen to be the current sheets and the reconnection sites. The particles can be energized by the electric field variations in these sites [10].

We are studying correlations between various quantities to further investigate this conjecture. Another preliminary study of stepping particles through a time snapshot of electric and magnetic fields from the hybrid code has been done. The particles that pass through a current sheet or a reconnection site get a jump in the magnetic moment. This increases the energy of the particle in the perpendicular direction. This study is being further pursued to step particles through the time evolving electric and magnetic fields outputted from the hybrid code. Another possible mechanism involved in the heating could be the second order Fermi acceleration of the protons.

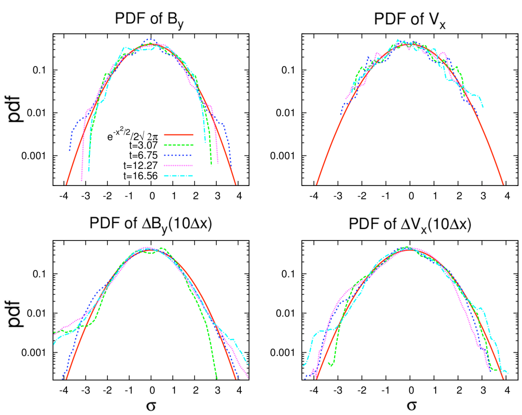

FIGURE 5. The probability density functions (PDFs) of magnetic field and velocity follow the Gaussians very nicely. The increments show tails, however, indicating strong intermittent structures.

CONCLUSIONS

This highly nonlinear system shows no obvious signs of significant wave activity but still dissipates energy and heats the ions preferentially. This suggests that we do not need to have strong wave modes and resonant wave particle interactions to heat ions in a collisionless plasma. One possible mechanism for heating is through energization of particles by the electric field variations in the intermittent reconnection sites and current sheets. Presently we are examining the correlation of particle energization with proximity to current sheets and reconnection sites to further investigate this conjecture. Another possibility is the stochastic second order Fermi heating of particles which will also be investigated in near future.

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