

## A MODEL FOR SPONTANEOUS ONSET OF FAST MAGNETIC RECONNECTION

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### ABSTRACT

We present a model for the spontaneous onset of fast magnetic reconnection in a weakly collisional plasma, such as the solar corona. When sheared magnetic fields in the corona undergo collisional (Sweet-Parker) reconnection, a narrow dissipation region forms around the X-line. This dissipation region dynamically becomes narrower during the reconnection process as stronger magnetic fields are convected toward the X-line. When the dissipation region becomes thinner than the ion skin depth, resistive magnetohydrodynamics breaks down as the Hall effect becomes important, and the Sweet-Parker solution ceases to exist. A transition to collisionless (Hall) reconnection ensues, increasing the reconnection rate by many orders of magnitude in a very short time. Predictions of the model are consistent with constraints set by observations of solar flares.

*Subject headings:* magnetic fields — plasmas — Sun: corona — Sun: coronal mass ejections (CMEs) — Sun: flares

*Online material:* color figure

### 1. INTRODUCTION

Magnetic reconnection is the driver of explosions in the solar corona. The first self-consistent description of magnetic reconnection, the Sweet-Parker model (Sweet 1958; Parker 1957), was immediately recognized as yielding energy release rates that are too slow to explain observations. Petschek reconnection (Petschek 1964) and related models (Priest & Forbes 1986) can be much faster, but they require anomalous resistivity (Sato & Hayashi 1979; Biskamp 1986), a process that is not well understood. In the past 10–15 yr, a new paradigm of collisionless (Hall) reconnection has emerged, in which nonmagnetohydrodynamic terms make reconnection much faster (Birn et al. 2001), about 6 orders of magnitude faster for solar flare parameters. Signatures of Hall reconnection have been observed in magnetospheric observations (Øieroset et al. 2001; Mozer et al. 2002) and laboratory experiments (Cothran et al. 2003; Ren et al. 2005).

However, explaining observed reconnection rates is only part of the problem. One must also explain why reconnection is explosive: explaining how magnetic energy can accumulate without significant dissipation and identifying the mechanism triggering the onset of fast (Hall) reconnection to release the stored energy are long-standing problems.

A self-consistent theoretical explanation for the onset of Hall magnetic reconnection has been elusive. One approach (Bhattacharjee et al. 2005) describes the onset as a smooth, but rapid, increase in the reconnection rate. More recently, Cassak et al. (2005) showed that Hall reconnection begins catastrophically as a transition from Sweet-Parker reconnection, which occurs when the thickness  $\delta$  of the Sweet-Parker dissipation region falls below the ion skin depth  $d_i = c/\omega_{pi}$ , where  $\omega_{pi} = (4\pi ne^2/m_i)^{1/2}$  is the ion plasma frequency and  $n$  is the plasma density. This is the length scale at which magnetohydrodynamics (MHD) breaks down and the Hall term in the generalized Ohm's law becomes important. While both the Sweet-Parker and Hall solutions are valid when  $\delta > d_i$ , the Sweet-Parker solution ceases to exist for  $\delta < d_i$ . As  $\delta$  falls below  $d_i$  during

Sweet-Parker reconnection, an abrupt transition to Hall reconnection ensues.

The catastrophic transition to Hall reconnection when  $\delta \sim d_i$  was demonstrated by (externally) decreasing the resistivity  $\eta$ . However, the idea that a solar eruption is caused by a change in the resistivity requires ad hoc assumptions about the poorly understood energetics of the corona. In this Letter, we suggest that the transition can occur as the result of the dissipation region thinning due to the convection of stronger magnetic fields into the dissipation region during slow Sweet-Parker reconnection. This is a generic process that is entirely self-driven: it relies on no external forcing or fine-tuning of any parameters. To our knowledge, this is the first self-consistent model for the spontaneous onset of fast reconnection.

To see why transitions to fast reconnection are generic to the reconnection process, consider a macroscopic equilibrium current layer with a small but nonnegligible resistivity. Without small-scale structure, the Hall term in Ohm's law is unimportant, so the system undergoes Sweet-Parker reconnection. Since the resistivity is small, the dissipation region is embedded within the macroscopic current layer. The thickness  $\delta$  of the Sweet-Parker current sheet is given by (Sweet 1958; Parker 1957)

$$\frac{\delta}{L} \sim \left( \frac{\eta c^2}{4\pi c_{A, \text{up}} L} \right)^{1/2}, \quad (1)$$

where  $c_{A, \text{up}} = B_{\text{up}}/(4\pi m_i n)^{1/2}$  is the Alfvén speed evaluated using the magnetic field  $B_{\text{up}}$  just upstream of the dissipation region and  $L$  is the macroscopic length of the Sweet-Parker current sheet in the outflow direction. During reconnection, stronger magnetic field lines are convected into the dissipation region by the inflow, which, according to equation (1), causes the Sweet-Parker sheet thickness to become smaller. When the dissipation region becomes thinner than  $d_i$ , a transition ensues.

In the following section, we review the salient properties of Sweet-Parker and Hall reconnection. In § 3, we describe the numerical simulations and their results, and we discuss implications for the onset of solar flares in § 4.

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## 2. SWEET-PARKER AND HALL RECONNECTION

In the Sweet-Parker model (Sweet 1958; Parker 1957), a steady state is reached when the convective inflow of magnetic field lines is balanced by diffusion of field lines toward the X-line,

$$\frac{v_{\text{in}}}{\delta} \sim \frac{\eta c^2}{4\pi\delta^2}, \quad (2)$$

where  $v_{\text{in}}$  is the inflow speed. From continuity,  $v_{\text{in}}L \sim v_{\text{out}}\delta$ , where  $v_{\text{out}}$  is the outflow speed. Therefore, the inflow Alfvén Mach number  $M_A = v_{\text{in}}/c_{A,\text{up}}$  (a measure of the reconnection rate) is given in the right-hand side of equation (1) since  $v_{\text{out}} \sim c_{A,\text{up}}$ . The extreme elongation of the dissipation region due to  $\eta$  being very small and  $L$  being macroscopic throttles Sweet-Parker reconnection. For solar flares, the Sweet-Parker prediction of  $M_A \sim 2 \times 10^{-7}$  is far smaller than inferred from observations, where we used typical parameters of  $n \sim 3 \times 10^9 \text{ cm}^{-3}$  for the preflare density,  $B_0 \sim 100 \text{ G}$  for the preflare coronal magnetic field, and a classical resistivity of  $\eta \sim 2 \times 10^{-16} \text{ s}$  based on a temperature of  $T \sim 10^6 \text{ K}$  (Priest & Forbes 2002). In the Sweet-Parker model,  $L$  is linked to the macrostructure of the coronal magnetic fields, which we have taken as  $L \sim 10^4 \text{ km}$  (Priest & Forbes 2002).

The physics of Hall reconnection is fundamentally different from that of Sweet-Parker. The motion of ions decouples from that of the electrons and the magnetic field at a distance  $d_i$  from the X-line. The electrons remain frozen-in to the magnetic field down to the electron skin depth  $d_e = c/\omega_{pe}$ . Where the species are decoupled, the Hall term in Ohm's law introduces whistler and/or kinetic Alfvén waves (depending on the plasma  $\beta$ ) into the system (Mandt et al. 1994; Rogers et al. 2001). Both waves are dispersive with  $\omega \propto k^2$ . The dispersive property of these waves causes the outflow jet from the X-line to open as discussed by Petschek (Rogers et al. 2001). In the absence of dispersive waves, reconnection is slow, as in the Sweet-Parker model.

Numerical simulations (Shay et al. 1999; Huba & Rudakov 2004; Shay et al. 2004) have shown that the inflow speed for steady state Hall reconnection is

$$v_{\text{in}} \sim 0.1c_{A,\text{up}} \quad (3)$$

(i.e.,  $M_A \sim 0.1$ ). This result has been found to be independent of electron mass (Shay & Drake 1998; Hesse et al. 1999), system size (Shay et al. 1999), and dissipation mechanism (Birn et al. 2001). Thus, we expect a dramatic increase in the reconnection rate when a transition from Sweet-Parker to Hall reconnection occurs.

## 3. NUMERICAL SIMULATIONS AND RESULTS

We perform numerical simulations using the massively parallel compressible two-fluid code F3D (Shay et al. 2004) in a periodic two-dimensional domain. The initial equilibrium magnetic field is a double current sheet configuration given by one period of a cosine sheet,  $B_{x,0}(y) = B_0 \cos(2\pi y/L_y)$ , where  $L_y$  is the size of the domain in the inflow direction, with pressure balance enforced by a nonuniform density profile,  $n(y) = n_0 + (B_0^2/8\pi T_0) \sin^2(2\pi y/L_y)$ . Here,  $n_0$  is a constant corresponding to the density at the edge of the domain, and  $T_0 = B_0^2/4\pi n_0$  is the temperature, which is assumed to be constant and uniform for simplicity. Therefore, the initial density at the center of the current sheet is  $1.5n_0$ . We impose no initial guide field. Lengths are

normalized to the ion skin depth  $d_{i,0}$  based on the density  $n_0$  at the edge of the computational domain, not the center of the X-line, which we denote as  $d_{i,x}$ . Magnetic field strengths, velocities, times, and resistivities are normalized to  $B_0$ , the Alfvén speed  $c_{A,0}$  based on  $B_0$  and  $n_0$ , the ion-cyclotron time  $\Omega_{ci}^{-1} = (eB_0/m_i c)^{-1}$ , and  $\eta_0 = 4\pi c_{A0} d_{i,0}/c^2$ , respectively.

The computational domain is of size  $L_x \times L_y = 409.6d_{i,0} \times 204.8d_{i,0}$  with a cell size of  $0.1d_{i,0} \times 0.1d_{i,0}$ . There is no viscosity, but fourth-order diffusion with coefficient  $2 \times 10^{-5}$  is used in all of the equations to damp noise at the grid scale. An electron mass of  $m_e = m_i/25$  is used. Although this value is unrealistic, the electron mass only controls dissipation at the electron scales, and this does not impact the rate of Hall reconnection. A small coherent perturbation  $\mathbf{B}_1 = -(0.004B_0L_y/2\pi)\hat{z} \times \nabla[\sin(2\pi x/L_x)\sin^2(2\pi y/L_y)]$  is used to initiate reconnection. The resistivity is taken to be uniform. Simulations are performed with  $\eta = 0.0025\eta_0$  and  $0.0090\eta_0$ , both of which exhibit transitions to fast reconnection. We present results from the  $\eta = 0.0025\eta_0$  simulation, which was initialized from the  $\eta = 0.0090\eta_0$  simulation at  $t = 5.364 \text{ k}\Omega_{ci}^{-1}$ . Initializing the simulation in this way introduces transient behavior, but it dies away (by  $t \sim 11 \text{ k}\Omega_{ci}^{-1}$ ) before small-scale dynamics become important.

When the system is evolved in time, the Hall effect is initially very small because the thickness of the current layer  $L_y/2 = 102.4d_{i,0}$  is large compared to  $d_{i,x}$ , so the system evolves essentially as it would in pure resistive MHD. A Sweet-Parker current sheet develops, as we will demonstrate later. The ion and electron inflow velocities, measured as the maximum value of the inflow into the X-line for each species, are plotted as a function of time late in the simulation in Figure 1a. Up until  $t \sim 18 \text{ k}\Omega_{ci}^{-1}$ , the electrons and ions are coupled as expected in MHD. The inflow speed is very small but is slowly rising due to a gradual increase in the upstream magnetic field strength  $B_{\text{up}}$ , as stronger magnetic fields are convected into the dissipation region. Figure 1b shows the slow increase in  $B_{\text{up}}$ , measured just upstream of the current sheet in the simulation.

When the ions decouple from the electrons, the inflow speeds begin to increase dramatically, and the system begins a transition to Hall reconnection. This transition is initiated when the thickness of the current sheet  $\delta$  falls below  $d_{i,x}$ , as is shown in Figure 1c. The thick solid line is  $d_{i,x}$  as a function of time. After decoupling, one must distinguish between the electron and ion current sheet thicknesses, which we denote as  $\delta_e$  and  $\delta_i$ , respectively. The thin solid line is  $\delta_e$ , determined by the half-width at half-maximum of the total current sheet. The dashed line is  $\delta_i$ , determined by the greater of  $\delta_e$  and the half-width at half-maximum of the total inflow current. The latter becomes nonzero where the electrons and ions decouple, and is therefore a measure of the edge of the ion dissipation region. One can see  $\delta_i$  decreasing from large scales (larger than  $d_{i,x}$ ) as the upstream magnetic field increases, and the transition begins when it is of the order of  $d_{i,x}$ .

Finally, to verify that the system is undergoing Sweet-Parker reconnection before the transition, and Hall reconnection after, we must check the validity of the inflow speed predictions from equations (2) and (3). The thick solid line of Figure 1d shows  $v_{\text{in}}$  as a function of time. The dashed line is the Sweet-Parker prediction from equation (2) ( $v_{\text{in}} \sim \eta/\delta$  in code units), while the thin solid line is the Hall reconnection prediction with a constant coefficient of 0.17, which is of the order of  $\sim 0.10$  as expected from equation (3). Clearly, up until about  $t \sim 18 \text{ k}\Omega_{ci}^{-1}$ , there is excellent agreement with the Sweet-Parker result. A gray-scale plot of the current sheet during the Sweet-Parker phase (at  $t = 11.4 \text{ k}\Omega_{ci}^{-1}$ ) is shown in Figure 2a, showing the characteristic

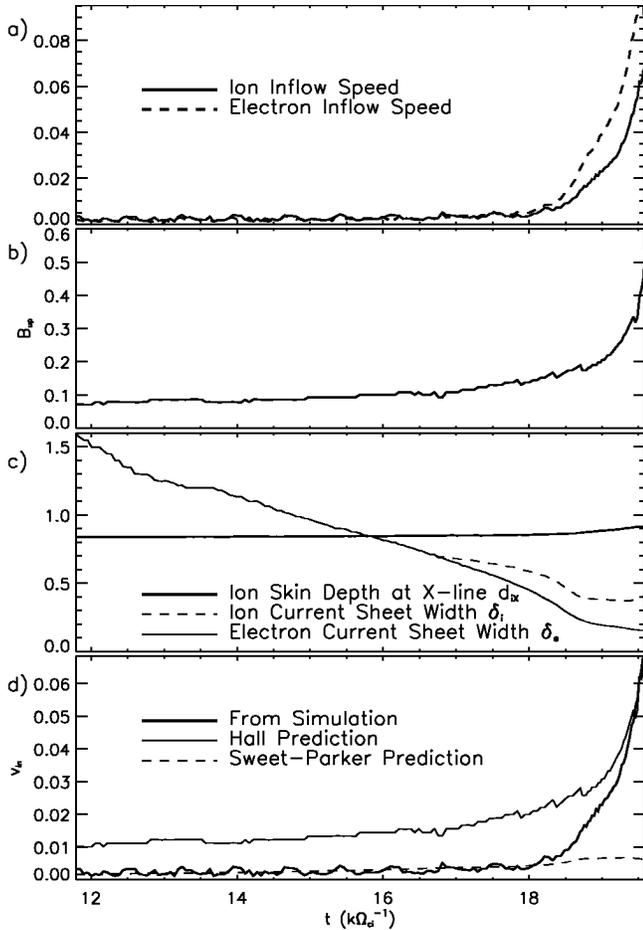


FIG. 1.—Time dependence (in thousands of ion-cyclotron times) of the (a) ion (thick solid line) and electron (thick dashed line) inflow velocities, (b) upstream magnetic field strength  $B_{up}$ , (c) electron (thin solid line) and ion (thin dashed line) current sheet thicknesses  $\delta_e$  and  $\delta_i$ , and ion skin depth (thick solid line) at the X-line  $d_{i,x}$ , and (d) ion inflow velocity from the simulation (thick solid line), with Sweet-Parker theory (thin dashed line, from eq. [2]) and Hall theory (thin solid line, from eq. [3] with 0.17 replacing 0.10).

elongated dissipation region (similar to those observed with pure MHD simulations by Jemella et al. 2004). After a relatively brief transition time lasting until  $t \sim 19.5 \text{ k}\Omega_{ci}^{-1}$ , the inflow speed is well modeled by the Hall prediction. A gray-scale plot of the current sheet during the Hall phase (at  $t = 19.6 \text{ k}\Omega_{ci}^{-1}$ ) is shown in Figure 2b, illustrating the open outflow configuration characteristic of Hall reconnection. We observe a large enhancement of the quadrupolar structure in the out-of-plane magnetic field, a signature of Hall reconnection (Mandt et al. 1994). Cuts across the current sheet at the X-line normalized to its maximum value are plotted as the dashed line and the dot-dashed line in Figure 2c, showing that  $\delta_e$  falls to  $d_e = 0.2d_i$  during Hall reconnection, as is expected when electron inertia provides the dissipation. For comparison, the solid line is a cut across the initial equilibrium current sheet.

#### 4. DISCUSSION

The spontaneous onset model presented here provides a possible explanation as to why reconnection sites in weakly collisional plasmas are apparently quiet for a long time as magnetic energy accumulates before a sudden onset of fast magnetic reconnection releases it. A rigorous comparison of this model with flare observations is challenging because the dissipation regions associated with the transition to fast reconnection are

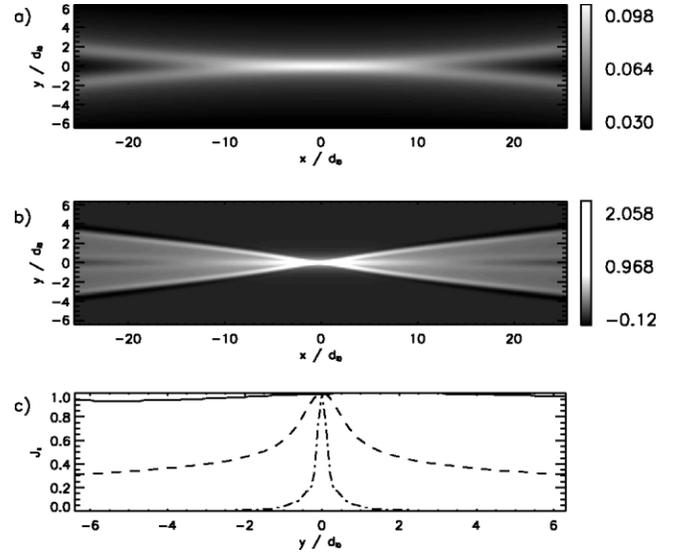


FIG. 2.—Gray-scale plot of the current sheet during (a) Sweet-Parker reconnection (at  $t = 11.4 \text{ k}\Omega_{ci}^{-1}$ ) and (b) Hall reconnection (at  $t = 19.6 \text{ k}\Omega_{ci}^{-1}$ ). (c) Cuts across the X-line for the same two sheets (dashed and dot-dashed lines, respectively) normalized to its maximum value. The initial current sheet profile is the solid line. Notice the color table for panel b) has been skewed for greater contrast, and the amplitude of the current density is vastly different for the two sheets. [See the electronic edition of the Journal for a color version of this figure.]

much narrower than can be resolved with satellite or ground-based observations. We can, however, compare some basic predictions with observations.

Taking  $\eta$  as a given, what is the critical upstream magnetic field strength  $B_*$  that would make the thickness of a Sweet-Parker current sheet equal to  $d_i$ ? Setting  $\delta = d_i$  in equation (1), we find

$$B_* \sim \sqrt{4\pi m_i n_0} \left( \frac{\eta c^2}{4\pi d_i^2} L \right) \sim 27 \text{ G} \quad (4)$$

using the values from § 2. This is accessible during reconnection in the corona.

What is the timescale for the quiet time  $\tau_q$  during which Sweet-Parker reconnection could be active but during which magnetic energy could accumulate? Since the field is frozen-in outside of the dissipation region, it is the time it takes for a field of strength  $B_*$  to be convected in by the inflow,

$$\tau_q = \int \frac{d\xi}{v_{in}}, \quad (5)$$

where  $\xi$  is the distance upstream from the X-line. This can be approximated using  $v_{in} \sim (\eta c^2 / 4\pi c_{A,0} L)^{1/2}$  and by assuming a linear profile in the magnetic field  $B = B_0 \xi / W_s$  in  $c_{A,0}$ , where  $W_s$  is the magnetic shear length, the characteristic scale length over which coronal magnetic fields change their direction. At present, measuring  $W_s$  in the corona is not possible, although based on observations Dahlburg et al. (2005) put an upper bound of 100 km. Integrating from  $\xi \sim B_* W_s / B_0$  to approximately zero gives

$$\tau_q \sim 2W_s \sqrt{\frac{4\pi L}{\eta c^2 c_{A,0} B_0}} \sim \left( \frac{W_s}{100 \text{ km}} \right) 1.2 \times 10^5 \text{ s}, \quad (6)$$

where  $c_{A,0}$  is the Alfvén speed based on  $B_0$ . The numerical factor is about 34 hr, which is a reasonable timescale for the

accumulation of magnetic energy due to footpoint motion in the photosphere (Dahlburg et al. 2005). Broader initial current layers would, of course, require a longer time before onset.

The time it takes for the transition from Sweet-Parker to Hall reconnection, corresponding to the time from onset until maximum flare signal, can be bounded above by the convective time across the Sweet-Parker current sheet  $\delta/v_{\text{in}}$ , which at the transition time is the same as the resistive time across the sheet  $(\eta c^2/4\pi\delta^2)^{-1}$  and the convective time along the sheet  $L/v_{\text{out}}$ . For our simulation, the resistive time is  $\sim 400\Omega_{ci}^{-1}$ , which compares reasonably well with the observed time of the transition (see Fig. 1d). For solar flare parameters, the resistive time across the sheet is approximately 9 s, which is comparable to the onset times seen in flares (Priest & Forbes 2002). The predicted observable parameters are quite consistent with solar flare phenomena.

The present simulations do not include the effect of an out-of-plane (guide) field, the more generic configuration for magnetic reconnection. It was conjectured (Cassak et al. 2005) that the transition to fast reconnection in the presence of a guide field is also catastrophic but occurs when the thickness of the current sheet reaches the ion Larmor radius  $\rho_s = c_s/\Omega_{ci}$ , where  $c_s$  is the ion sound speed, instead of the ion skin depth  $d_i$ . This is because  $\rho_s$  is the scale where dispersive (kinetic Alfvén) waves become important in the presence of a guide field (Rogers et al. 2001). Interestingly, recent laboratory experiments at the Versatile Toroidal Facility (Egedal et al. 2000) have ob-

served spontaneous reconnection, and preliminary diagnostics suggest that the thickness of the current sheet at onset is very close to their value of the ion Larmor radius  $\rho_s$  (J. Egedal 2006, private communication).

Finally, Longcope et al. (2005) studied *TRACE* observations of an active region reconnecting with a nearby flux loop as it emerged from the corona. A phase of slow reconnection was observed for  $\sim 24$  hr, during which magnetic energy accumulated in the corona. This was followed by fast reconnection lasting  $\sim 3$  hr. The onset was sudden, with no visible trigger mechanism observed. The energy released during fast reconnection was shown to be comparable to the energy accumulated during slow reconnection. Based on parameters inferred from the observations ( $L \sim 3 \times 10^4$  km,  $n \sim 10^9$  cm $^{-3}$ , a loop voltage of  $10^9$  V, a separator length of  $2 \times 10^5$  km, and a sheet current of  $I \sim 1.34 \times 10^{11}$  A [Longcope et al. 2005]), the fast reconnection rate was  $M_A \sim 0.05$ , based on a reconnection electric field of  $E \sim 5$  V m $^{-1}$  and a reconnecting magnetic field of  $B_0 \sim 4$  G, consistent with Hall reconnection. These observations provide solid evidence for the accumulation of magnetic energy during a slow reconnection phase followed by the spontaneous onset of fast reconnection, as proposed here.

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